A CHARACTERIZATION OF HALF POSITIONAL ω-REGULAR LANGUAGES

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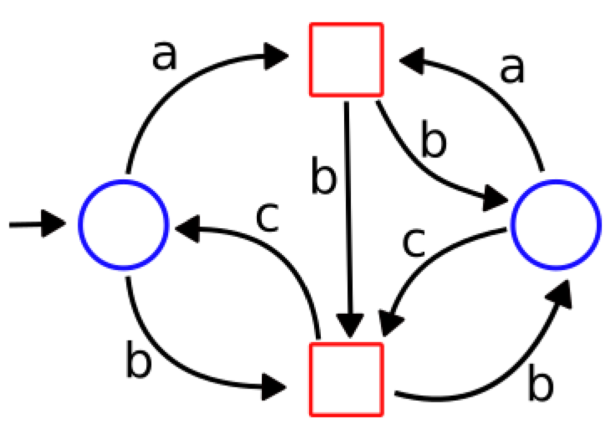
Games and Half Positionality

Games on Graphs:



Eve

Adam

- A play produces an infinite word $w \in \Sigma^\omega$.
- Winning objective: $L \subseteq \Sigma^\omega$.

Half Positionality:

- Positional strategy (for Eve): $\sigma\colon V_{\mathsf{Eve}} \to E$.
- A language L is **half positional** if, for every game $\mathcal{G}$ using L as winning condition:

Eve can win $\mathcal{G}$ $\implies$ She can win using a positional strategy.

Main Results

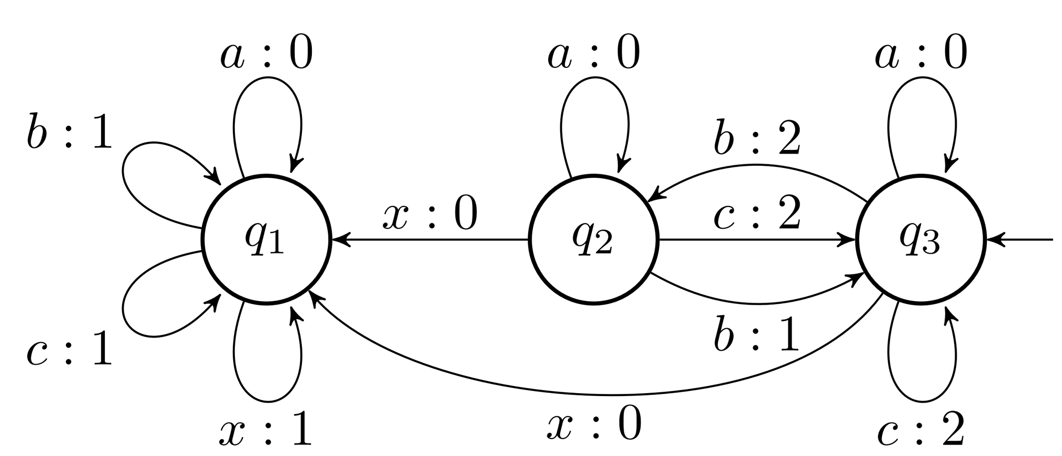
Complete characterization of deterministic parity automata recognizing half positional languages.

Corollaries for ω-regular languages:

- Decidability in polynomial time.
- Half positional for finite, Eve-games $\iff$ Half positional for all games
- Closure of prefix-independent half positional languages under union (Kopczyński's conj. for ω-reg).
- Closure of half positional languages under addition of a neutral letter (Ohlmann's conj. for ω-reg).

The Characterization: An Example

$$L \;=\; \mathsf{Inf}(a) \;\text{ or }\; (\mathsf{No}(x) \text{ and } \neg\mathsf{Inf}(bb))\,,$$
over $\Sigma = \{a, b, c, x\}$.



Parity automaton $\mathcal{A}$

Total order over residuals

Residuals totally ordered by inclusion.

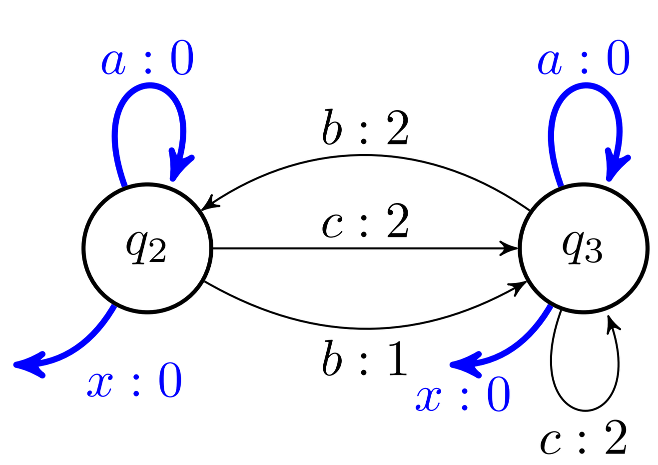
$$x^{-1}L = \mathsf{Inf}(a) \;\subsetneq\; \varepsilon^{-1}L = L$$

States of $x^{-1}L$ ➔ q_1

States of $\varepsilon^{-1}L$ ➔ $q_2 \sim q_3$

Uniformity of 0-transitions

If $q \sim p$:
$$q \xrightarrow{a:0} \;\implies\; p \xrightarrow{a:0}$$

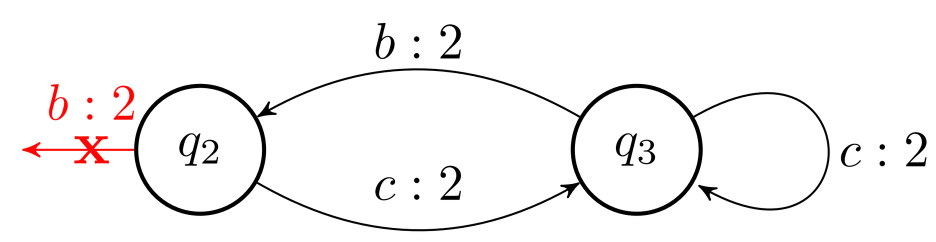


Total order of 1-safe languages

$\mathcal{A}|_{\geq 1} =$ Restriction to priorities ≥ 1

$\mathcal{L}_1(q) = \{w \mid q \overset{w}{\rightsquigarrow} \text{ avoids 1 in } \mathcal{A}|_{\geq 1}\}$

Inside each SCC of $\mathcal{A}|_{\geq 1}$, inclusion of $\mathcal{L}_1(q)$ is a total preorder.



$$\mathcal{L}_1(q_2) = c^* + (c^+b)^* \;\subsetneq$$
$$\subsetneq\; \mathcal{L}_1(q_3) = c^* + (bc^+)^*$$