On the size of good-for-games Rabin automata and its link with the memory in Muller games

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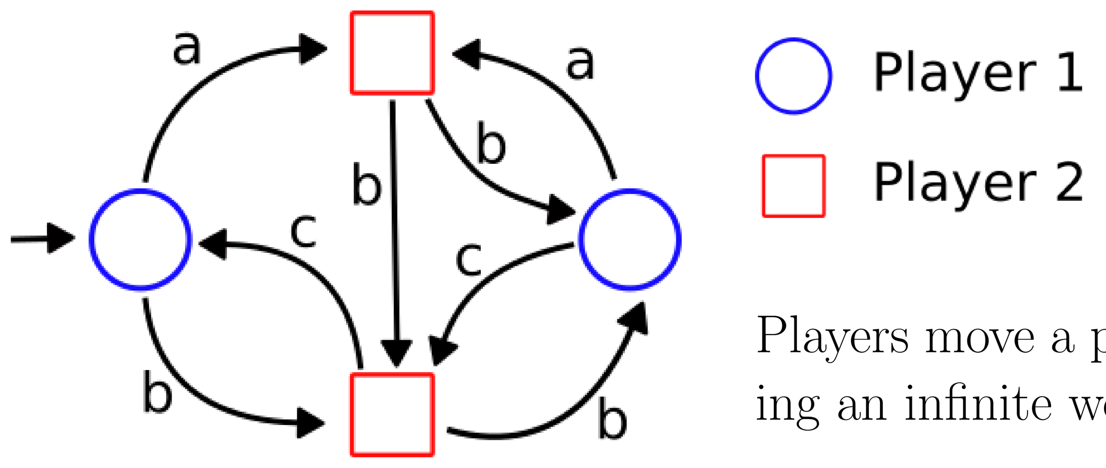
Muller games



Players move a pebble producing an infinite word $w \in C^\omega$.

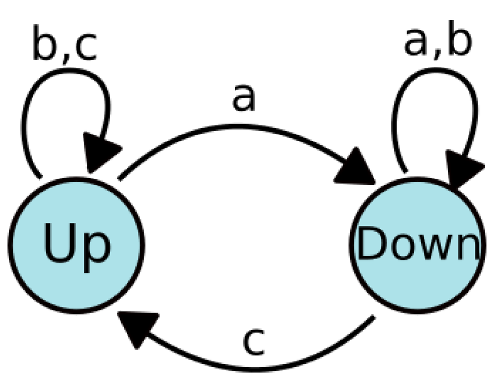
Muller languages:

For $\mathcal{F}$ a family of subsets of colours (for ex. $\mathcal{F} = \{\{a,b,c\},\{a\},\{b\}\}$):

$\mathcal{L}_\mathcal{F} = \{w \in C^\omega \mid$ Colours appearing infinitely often in w form a set in $\mathcal{F}\}$.

Player 1 wins the Muller game if the word $w \in C^\omega$ produced is in $\mathcal{L}_\mathcal{F}$.

Memory structures



Structure that tells Player 1 how to play.

It is updated after each move in the game.

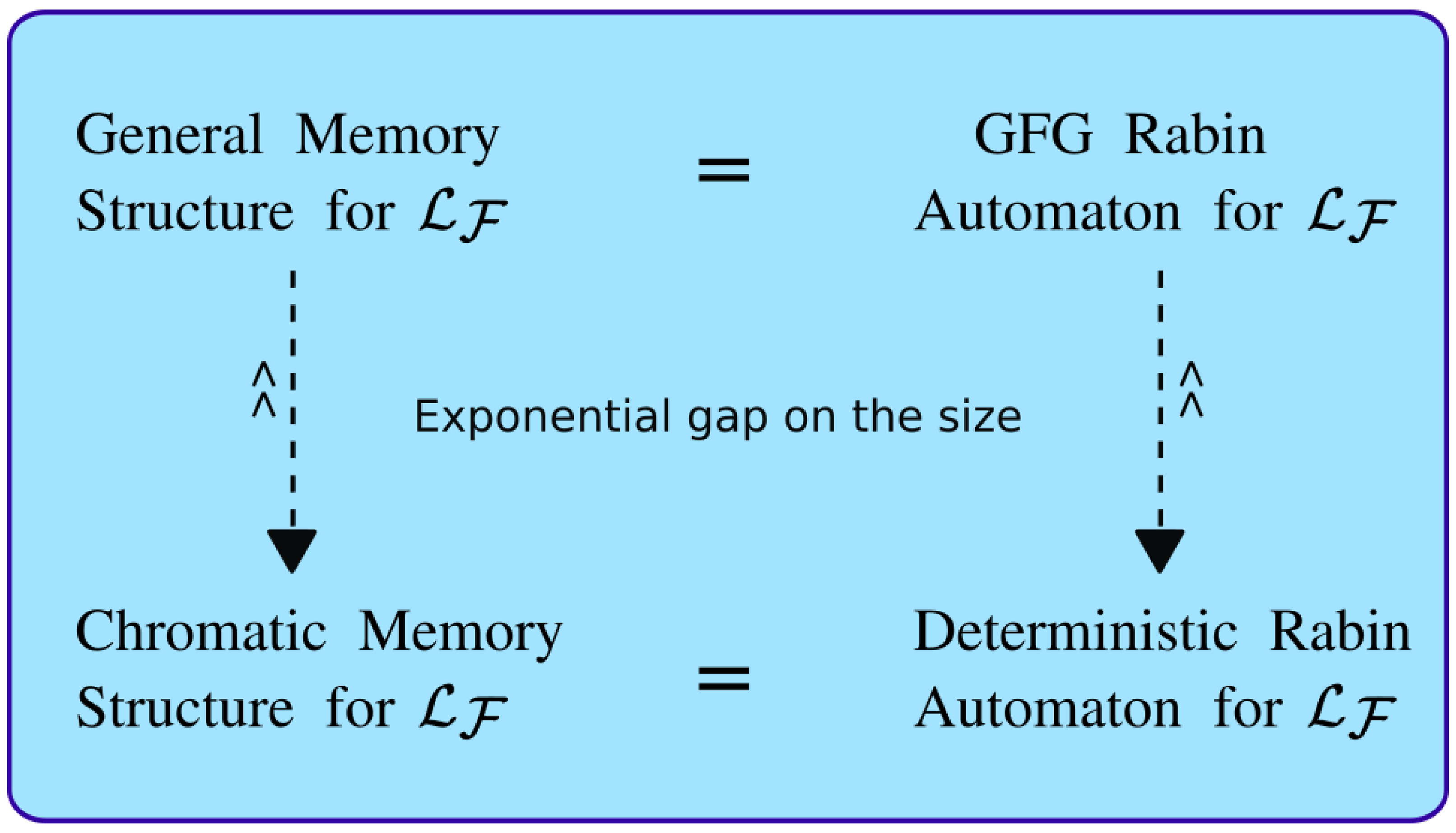
Two kinds of memory structures:

General memory: Update transitions can depend on the specific edges of the game.

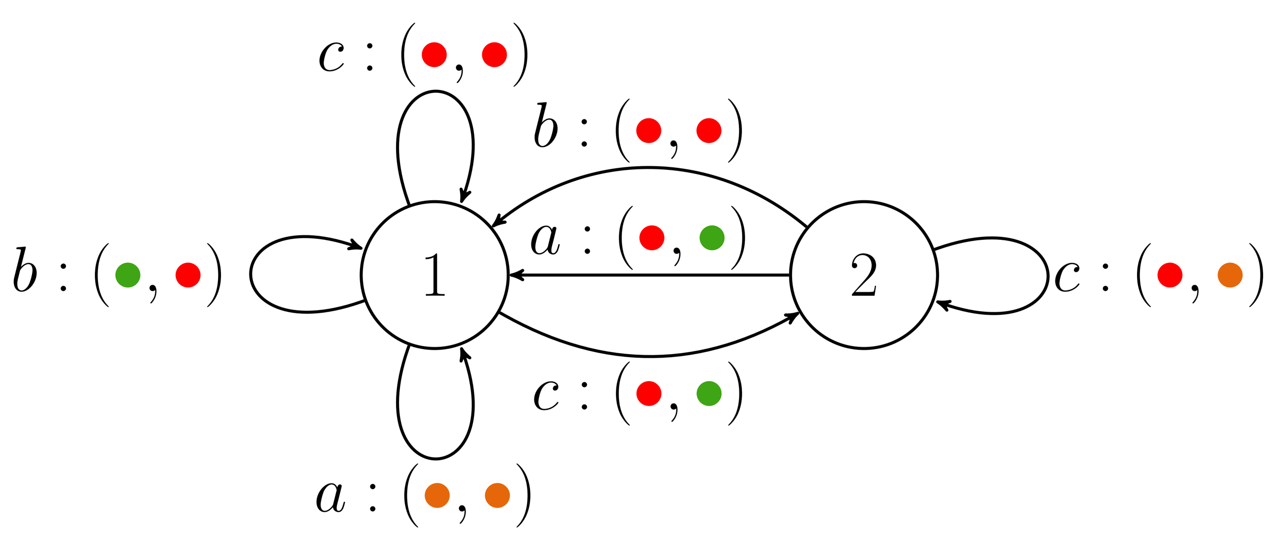
Chromatic memory: Update transitions only depend on the colours of the condition (for example, the memory structure above).

Main results: Correspondance Memory $\leftrightarrow$ Automata

Let $\mathcal{L}_\mathcal{F}$ be a Muller language.



Rabin automata



Automaton recognising $\mathcal{L}_\mathcal{F}$ for $\mathcal{F} = \{\{a,b\},\{a,c\},\{b\}\}$.

Rabin condition:

Output alphabet: $\Gamma = \{\bullet, \bullet, \bullet\}^k$, ($k = 2$ in the example). A run in $\mathcal{A}$ produces an infinite sequence of arrays $v_1 v_2 v_3 \cdots \in \Gamma^\omega$.

A run is accepting if for some component $x \in \{1,\dots,k\}$, $v_i[x] = \bullet$ infinitely often and $v_i[x] \neq \bullet$ from some point onwards.

Good-for-Gameness (GFG)

A non-deterministic automaton $\mathcal{A}$ is *good-for-games* (GFG) if there exists a strategy resolving its non-determinism

$$\sigma : \Sigma^* \to \Delta,$$

such that:

$w \in \Sigma^\omega$ is accepted by the automaton
$\iff$
The run over w obtained following σ is accepting.

$\Sigma =$ Input alphabet. $\Delta =$ Transitions of the automaton.

For example, the Rabin automaton on the left is GFG (but it is not deterministic).