On the Minimisation of Transition-Based Rabin Automata and the Chromatic Memory Requirements of Muller Conditions

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Minimisation of Automata

State of the art minimisation of co-Büchi automata:

	State-based	Transition-based
Deterministic	`NP`-complete (Schewe 2010)	**?**
Good-For-Games	`NP`-complete (Schewe 2020)	Polynomial (Abu Radi-Kupferman 2019)

Our contributions:

Theorem

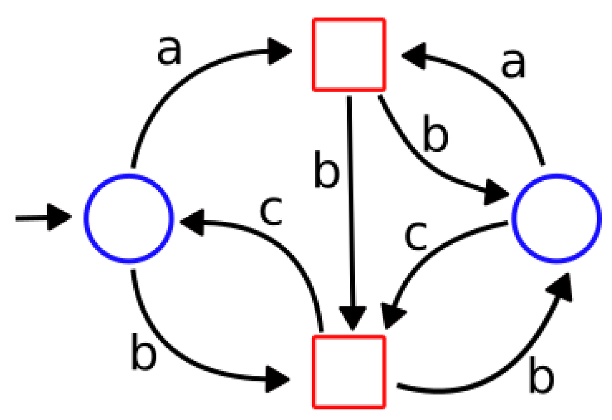
Minimisation of transition-based Rabin automata is `NP`-complete.

Remark 1. *For the reduction we use a language that is a* Muller condition*: whether a word $w \in \Sigma^\omega$ belongs to the language only depends on $Inf(w)$.*

Proposition

Minimisation of parity automata recognising Muller conditions can be done in polynomial time.

Muller games



Player 1

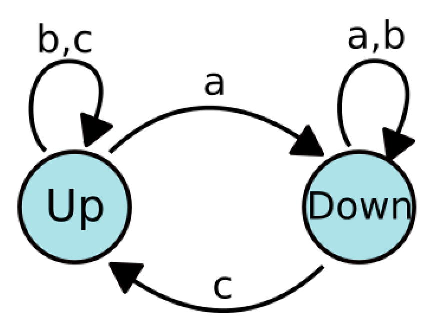
Player 2

Players move a pebble in turns producing an infinite word $w \in \{a, b, c\}^\omega$.

Muller condition: Family of subsets of colours $\mathcal{F} = \{\{a, b, c\}, \{a\}, \{b\}\}$.

Player 1 wins if the set of colours produced infinitely often is in $\mathcal{F}$.

Memory structures



Structure that tells Player 1 how to play.
It is updated after each move in the game.

3 kinds of memory structures:

General memory: Update transitions can depend on the specific edges of the game.

Chromatic memory: Update transitions only depend on the colours of the condition (for example, the memory structure above).

Arena-independent memory: Memory structure that can be used in any game where the winning condition is $\mathcal{F}$.

Question (Kopczyński 2006)

Is it true that, if Player 1 wins a game using a given winning condition, she requires the same amount of "general memory" than "chromatic memory" to win it?

Answer

No. There are Muller conditions for which Player 1 requires strictly more "chromatic memory" than "general memory" to win.

Memory and Rabin automata

In the following, "automaton" stands for "deterministic transition-based automaton".

Theorem

Let $\mathcal{F}$ be a Muller condition. Any Rabin automaton recognising $\mathcal{F}$ can be used as an arena-independent memory for $\mathcal{F}$-games. Conversely, any arena-independent memory for $\mathcal{F}$-games can be interpreted as a Rabin automaton recognising $\mathcal{F}$. In particular, the following quantities coincide:

- The size of a minimal Rabin automaton recognising $\mathcal{F}$.
- The smallest amount of chromatic memory required to win $\mathcal{F}$-games.
- The size of a minimal arena-independent memory for $\mathcal{F}$-games.

Corollary

Determining the amount of chromatic memory needed in games using a Muller condition $\mathcal{F}$ is `NP`-complete, even if the condition is represented by its Zielonka tree.

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